

Chapter 2

Exercise 2A

- 1 a $\frac{\sqrt{3}}{2}$
 b $\frac{1}{2}$
 c $\frac{\sqrt{3}}{2}$
 d 0
 e $-\frac{\sqrt{3}}{2}$
 f $-\frac{1}{\sqrt{2}}$
 g 1
 h $\frac{1}{2}$
 i 0
 j $-\frac{\sqrt{3}}{2}$
 k $-\frac{1}{\sqrt{3}}$
 l $\frac{\sqrt{3}}{2}$
 m 0
 n $\frac{1}{\sqrt{3}}$
 o $\frac{1}{2}$
 p 0
- 2 $\sin A = \frac{3}{\sqrt{13}}$ $\cos A = \frac{2}{\sqrt{13}}$
 3 $\cos P = \frac{\sqrt{11}}{6}$ $\tan P = \frac{5}{\sqrt{11}}$
 4 a $\cos X = \frac{12}{13}$ $\tan X = \frac{5}{12}$
 b $\cos X = -\frac{12}{13}$ $\tan X = -\frac{5}{12}$

Exercise 2B

- 1 a $\frac{\pi}{6}$
 b $\frac{5\pi}{6}$
 c $\frac{5\pi}{3}$
 d $\frac{3\pi}{4}$
 e $\frac{7\pi}{4}$
 f $\frac{2\pi}{3}$
 g $\frac{4\pi}{3}$
 h $\frac{5\pi}{2}$
 i $-\frac{\pi}{3}$
 j 4π

- 2 a 45°
 b 15°
 c 36°
 d 270°
 e 225°
 f 135°
 g 210°
 h 420°
 i -30°
 j 540°
- 3 a (i) 0.82 (ii) 5.65
 b (i) 249.8 (ii) 90.0
- 4 a $\frac{1}{\sqrt{2}}$
 b $-\frac{1}{\sqrt{2}}$
 c $-\frac{\sqrt{3}}{2}$
 d $-\frac{1}{\sqrt{3}}$
 e -1
 f $\sqrt{3}$
 g $\frac{1}{2}$
 h -1

Exercise 2c

- 1 a $\sin(P + Q)$
 b $\sin(M - N)$
 c $\cos(105^\circ)$
 d $\sin(-15^\circ) = -\sin(15^\circ)$
 e $\sin\left(\frac{2\pi}{15}\right)$
 f $\cos(205^\circ)$
 g $\cos(-15^\circ) = \cos(15^\circ)$
 h $\cos\left(\frac{11\pi}{12}\right)$
- 2 a $\sin P \cos Q + \cos P \sin Q$
 b $\cos R \cos S + \sin R \sin S$
 c $\cos 48 \cos A - \sin 48 \sin A$
 d $\cos 15 \sin B - \cos B \sin 15$
 e $\frac{1}{2}(\cos x + \sqrt{3} \sin x)$
 f $\frac{1}{\sqrt{2}}(\cos x + \sin x)$

$$\mathbf{g} \quad -\frac{1}{2}(\sin t + \sqrt{3} \cos t)$$

$$\mathbf{h} \quad -\frac{1}{2}(\sqrt{3} \cos x + \sin x)$$

$$\begin{aligned} \mathbf{3} \quad \mathbf{a} \quad & \sin 30 \cos t + \cos 30 \sin t \\ &= 2(\sin 30 \cos t - \cos 30 \sin t) \\ & \tan 30 \cos t + \sin t \\ &= 2 \tan 30 \cos t - 2 \sin t \\ & 3 \sin t = \tan 30 \cos t \\ & \tan t = \frac{1}{3} \tan 30 \\ & \tan t = \frac{1}{3\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & 2 \cos x \cos\left(\frac{\pi}{4}\right) - 2 \sin x \sin\left(\frac{\pi}{4}\right) \\ &= \cos\left(\frac{\pi}{4}\right) \cos x + \sin x \sin\left(\frac{\pi}{4}\right) \\ & 2 \cos x - 2 \sin x = \cos x + \sin x \\ & 3 \sin x = \cos x \\ & \tan x = \frac{1}{3} \end{aligned}$$

Exercise 2D

$$\mathbf{1} \quad \mathbf{a} \quad 0$$

$$\mathbf{b} \quad \frac{\sqrt{3}}{2}$$

$$\mathbf{c} \quad \frac{1}{2}$$

$$\mathbf{d} \quad -\frac{\sqrt{3}}{2}$$

$$\mathbf{e} \quad \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\mathbf{f} \quad \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\mathbf{2} \quad 0$$

$$\mathbf{3} \quad \frac{16}{65}$$

$$\mathbf{4} \quad \sin P = \frac{3}{5}$$

$$\cos P = \frac{4}{5}$$

$$\sin Q = \frac{1}{\sqrt{50}}$$

$$\cos Q = \frac{7}{\sqrt{50}}$$

$$\begin{aligned} \cos P \cos Q - \sin P \sin Q &= -\frac{3}{25\sqrt{2}} + \frac{14\sqrt{2}}{25} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

$$\mathbf{5} \quad \cos X = \frac{3}{5}$$

$$\sin X = \frac{4}{5}$$

$$\begin{aligned} \sin X \cos 30 + \cos X \sin 30 &= \frac{3}{10} + \frac{2\sqrt{3}}{5} \\ &= \frac{1}{10}(3 + 4\sqrt{3}) \end{aligned}$$

$$\mathbf{6} \quad \sin A = \frac{2}{3}$$

$$\cos A = \frac{\sqrt{5}}{3}$$

$$\cos A \cos\left(\frac{3\pi}{2}\right) - \sin A \sin\left(\frac{3\pi}{2}\right) = \frac{2}{3}$$

$$\mathbf{7} \quad \tan P = \frac{2}{\sqrt{5}}$$

$$\tan Q = \frac{\sqrt{5}}{3}$$

$$\begin{aligned} \tan(P+Q) &= \frac{\tan P + \tan Q}{1 - \tan P \tan Q} \\ &= 3\left(\frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3}\right) = \frac{11}{\sqrt{5}} \end{aligned}$$

OR

$$\sin P = \frac{2}{3}$$

$$\cos P = \frac{\sqrt{5}}{3}$$

$$\sin Q = \frac{\sqrt{5}}{4}$$

$$\cos Q = \frac{3}{4}$$

$$\frac{\sin(P+Q)}{\cos(P+Q)} = \frac{\sin P \cos Q + \cos P \sin Q}{\cos P \cos Q - \sin P \sin Q} = \frac{11}{\sqrt{5}}$$

$$\mathbf{8} \quad \sin A = \frac{2}{\sqrt{13}}$$

$$\sin B = \frac{\sqrt{3}}{4}$$

$$\cos A = \frac{3}{\sqrt{13}}$$

$$\cos B = \frac{\sqrt{13}}{4}$$

$$\begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{1}{52}(26 + 3\sqrt{39}) \end{aligned}$$

9 Mark X vertically below B.

$$AX = 6$$

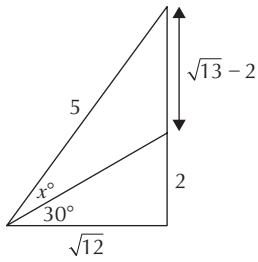
$$XC = 15$$

$$AC = 21$$

$$21^2 = 10^2 + 17^2 - 2 \times 10 \times 17 \cos ABC$$

$$\Rightarrow \cos ABC = -\frac{13}{85}$$

10



$$\begin{aligned}\cos x &= \frac{5^2 + 4^2 - (\sqrt{13} - 2)^2}{2 \times 4 \times 5} \\ &= \frac{6 + \sqrt{13}}{10}\end{aligned}$$

11 $\frac{2ab}{a^2 + b^2}$

Exercise 2E

1 a $\frac{\sqrt{3}}{2}$

b $\frac{1}{\sqrt{2}}$

c $\frac{1}{2}$

d $-\frac{1}{2}$

2 a $\frac{4\sqrt{21}}{25}$

b $-\frac{17}{25}$

c $-\frac{4\sqrt{21}}{17}$

3 $-\frac{120}{169}$

4 $-\frac{11}{5\sqrt{5}}$

5 $\frac{1}{50}(24 - 7\sqrt{3})$

6 $2\cos^2 A - 1 = \frac{3}{5}$

$$2\cos^2 A = \frac{8}{5}$$

$$\cos^2 A = \frac{4}{5}$$

$$\cos A = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

7 $\frac{3}{5}$

8 $\tan A = \frac{3}{4}$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{24}{7}$$

9 $-\frac{12}{13}$

10 7.937 m^2

11 a $180 = x + 2y$

$$y = 90 - \frac{x}{2}$$

$$\frac{b}{\sin x} = \frac{a}{\sin(90 - \frac{x}{2})}$$

b $b = \frac{2a \sin(\frac{x}{2}) \cos(\frac{x}{2})}{\cos(\frac{x}{2})} = 2a \sin(\frac{x}{2})$

Exercise 2F

1 a -1

b $\frac{1}{7}$

c $-\frac{3}{4}$

2 a 1

b $\frac{1}{7}$

c $\frac{4}{3}$

Exercise 2G

1 a $\sin(x + 45) = \frac{1}{\sqrt{2}}(\cos x + \sin x)$

$$\cos(x + 45) = \frac{1}{\sqrt{2}}(\cos x - \sin x)$$

adding gives $\sqrt{2} \cos x$

b $2\cos(x + 30) = (\sqrt{3} \cos x - \sin x)$

$$-\sin x = \sqrt{3} \cos x - 2\sin x$$

c $\sin(x - 60) = -\frac{1}{2}\sqrt{3} \cos x + \frac{1}{2} \sin x$

$$\cos(x + 30) = \frac{1}{2}\sqrt{3} \cos x - \frac{1}{2} \sin x$$

adding gives 0

d $\sin(x + 225) = -\frac{1}{\sqrt{2}}(\cos x + \sin x)$

$$\cos(x + 135) = -\frac{1}{\sqrt{2}}(\cos x + \sin x)$$

subtracting gives 0

e $a = \sin(\theta + \frac{\pi}{3}) = \frac{1}{2}(\sqrt{3} \cos \theta + \sin \theta)$

$$b = \cos(\theta + \frac{\pi}{6}) = \frac{1}{2}(\sqrt{3} \cos \theta - \sin \theta)$$

$$\sin \theta - a + b = 0$$

2 a

$$x = \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$y = \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$x + y = 2\cos\alpha\cos\beta$$

b

$$x = \sin(\alpha + \beta) = \cos\beta\sin\alpha + \cos\alpha\sin\beta$$

$$y = \sin(\alpha - \beta) = \cos\beta\sin\alpha - \cos\alpha\sin\beta$$

$$x - y = 2\cos\alpha\sin\beta$$

3 $\cos(2x) = 2\cos^2 x - 1$

$$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$

4 $\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}$

divide top and bottom by $\cos x \cos y$

$$= 1 - \tan x \tan y$$

5 a $(\cos x - \sin x)(\cos x + \sin x)$

$$= \cos^2 x - \sin^2 x = \cos(2x)$$

b $\cos(2x) = 2\cos^2 x - 1$

$$2\cos^2 x - 1 + \cos x$$

$$= 2\cos^2 x + \cos x - 1$$

$$= (2\cos x - 1)(\cos x + 1)$$

6 $2(\cos^2 x - \sin^2 x) - \cos^2 x$

$$\cos^2 x - 2\sin^2 x$$

$$1 - \sin^2 x - 2\sin^2 x = 1 - 3\sin^2 x$$

7 $\sin(2a + a)$

$$\sin(2a)\cos a + \cos(2a)\sin a$$

$$(2\sin a \cos a)\cos a + (\cos^2 a - \sin^2 a)\sin a$$

$$\sin a(2\cos^2 a + \cos^2 a - \sin^2 a)$$

$$\sin a(3\cos^2 a - \sin^2 a)$$

$$\sin a(3(1 - \sin^2 a) - \sin^2 a)$$

$$\sin a(3 - 4\sin^2 a)$$

$$3\sin a - 4\sin^3 a$$

8 $\cos 4\theta = \cos(2\theta + 2\theta)$

$$= 2\cos^2(2\theta) - 1$$

$$= 2(2\cos^2\theta - 1)^2 - 1$$

$$= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1$$

$$= 8\cos^4\theta - 8\cos^2\theta + 1$$

9 $(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$

$$= \cos^2 A - \sin^2 A = \cos(2A)$$

10 a $\sin(x + y) = \cos y \sin x + \cos x \sin y$

$$\sin(x - y) = \cos y \sin x - \cos x \sin y$$

adding gives $2\sin x \cos y$

b $x + y = A \Rightarrow x = \frac{A+B}{2}$

$$x - y = B \Rightarrow y = \frac{A-B}{2}$$

$$2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

c $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

d $\frac{2\sin(4x)\cos x}{2\cos(2x)\cos x} = \frac{\sin(4x)}{\cos(2x)}$

$$= \frac{2\sin(2x)\cos(2x)}{\cos(2x)} = 2\sin(2x)$$

Exercise 2H

1 a $k = 5, \alpha = 36.87^\circ$

b $k = \sqrt{34}$

$$\alpha = 329.04^\circ$$

c $k = \sqrt{5}$

$$\alpha = 116.57^\circ$$

2 a $k = \sqrt{2}$

$$\alpha = \frac{\pi}{4}$$

b $k = 2$

$$\alpha = \frac{7\pi}{6}$$

c $k = \sqrt{41}$

$$\alpha = 2.246$$

● ANSWERS

Exercise 2I

1 a $\sqrt{5} \cos(x - 63.43)$

b $\sqrt{5} \cos(x + 296.57)$

c $\sqrt{5} \sin(x + 26.57)$

d $\sqrt{5} \sin(x - 333.43)$

2 $\sqrt{41} \cos(x + 38.66)$

3 $2 \sin\left(\theta - \frac{7\pi}{4}\right)$

4 $\sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$

5 $2 \cos\left(x + \frac{\pi}{6}\right)$

6 $\sqrt{18} \sin\left(x + \frac{\pi}{4}\right)$

7 $5 \sin(2x - 36.87)$

8 $\sqrt{13} \cos(3x - 33.69)$

9 $2 \cos(x - 30)$